JEEVAN-KUSHALAIAH METHOD TO FIND THE COEFFICIENTS OF CHARACTERISTIC EQUATION OF A MATRIX AND INTRODUCTION OF SUMMETOR

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Abstract: The method is based on one of the arts of ancient tradition is Basket Weaving registered resolution number: 12011/68/93-BCC(C) and date 10/09/1993 Andhra Pradesh, India. The method gives maximum number of possible square sub matrices of a square matrix starting from minimum order to maximum order. By this method, coefficients of characteristic equation of a square matrix are calculated. The Summetor symbol is newly introduced. The function of Summetor is same as Factorial instead of multiplication, addition is performed. The method name is named on the authors Jeevan and Kushalaiah of the article or manuscript.

Keywords: [1] Basket Making, [2] Summetor, [3] J-K chart, [4] Formula of Coefficients

I. Introduction

Basket Weaving or Basketry or Basket Making[1] is an art of making thin strips of cane in to a basket or other similar form. Horizontal and Vertical strips are like rows and Columns of a matrix and coordinated position of row strips and column strips gives element position of the matrix.

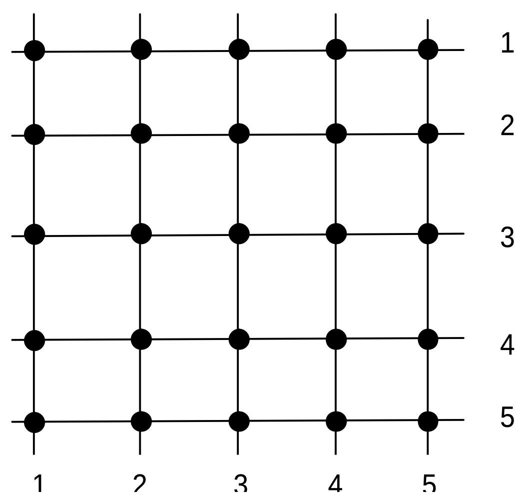


Fig.1 Basket maker making a Basket with strips of Cane, 5x5 matrix with 5 Horizontal strips and 5 vertical strips . The dots are the elements which are formed by Coordinates of horizontal and vertical strips

We know that

$$|\lambda I - A| = 0 \qquad \ldots\ldots\ldots\ldots\ldots \qquad (i)$$

The polynomial equation

$$C_0\lambda^{n} + C_1\lambda^{n-1} + C_2\lambda^{n-2} + \ldots\ldots C_{n-1}\lambda + C_n = 0 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad (ii)$$

SUMMETOR[2]: sum of variable element starting from minimum limit to maximum limit n-times

$$\sum_{P=1}^{n} \mathbf{P} = P!$$

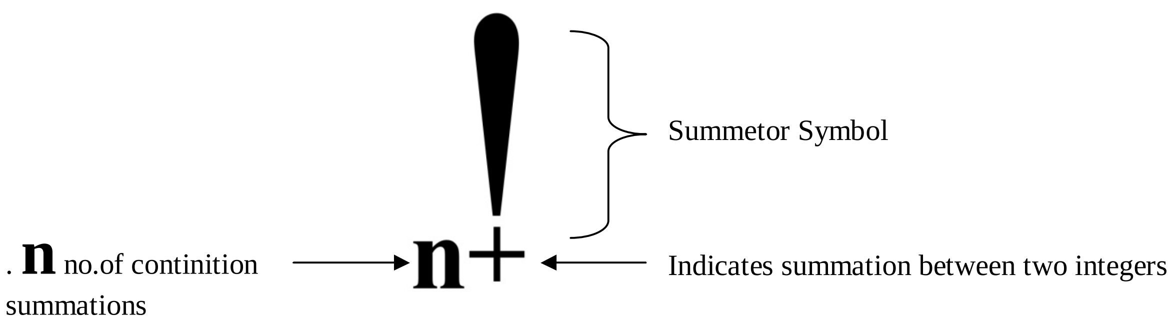


Fig 2. Summetor symbol

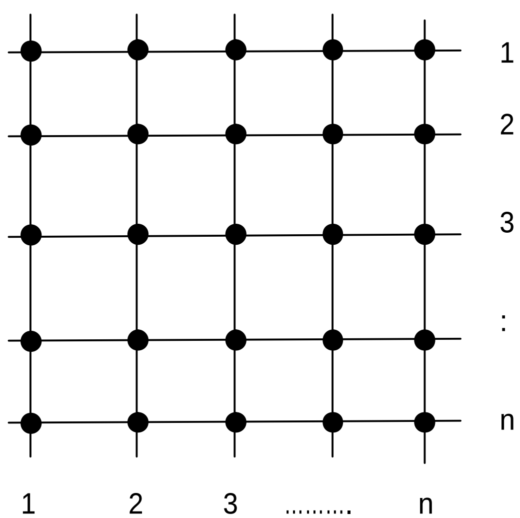
II. Calculation of Coefficients

Let us assume that a square matrix-A has n-rows and n- columns. The order of sub matrices is increases from [1x1] to [n-1 x n-1]. Every Coefficient is sum of determinant sub matrices to the order respectively.

Each determinant of sub matrix is denoted by $\Phi_{j,k}$

Where j - the order of the sub matrix

k - Sub matrix no.



$$= \begin{bmatrix} a11 & a12 & a13\ldots\ldots & a1n \\ a21 & a22 & a23\ldots\ldots & a2n \\ a31 & a32 & a33 \quad \ldots\ldots & a3n \\ \vdots & \vdots & \vdots & \vdots \\ an1 & an2 & an3\ldots\ldots & ann \end{bmatrix}$$

Fig.3 Formation of **nxn** Matrix with n horizontal and vertical strips of cane.

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The Important notes of Jeevan – Kushalaiah Method are given below

Notes .1: $C_0 = 1$ …. (ii.a)

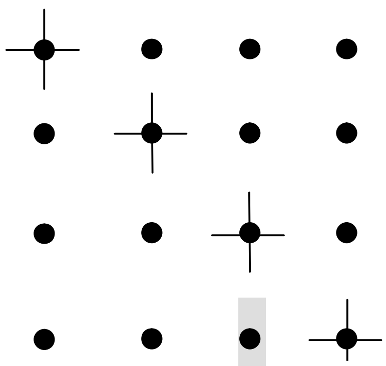
Notes .2: C_1 = Trace (A) = sum of diagonal elements = $\sum_{i=1}^{n} aii$ …. (ii.b)

Notes .3: C_n = Determinant of Matrix-A = $|\mathbf{A}|$ …. (ii.c)

Calculation of co-factor C_1

Put one horizontal and vertical strip on each diagonal element. Sum the all the determinants of coordinates of vertical and horizontal elements.

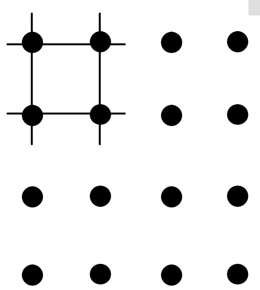
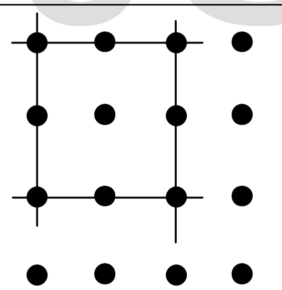
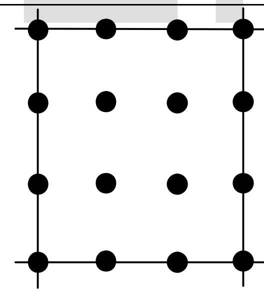
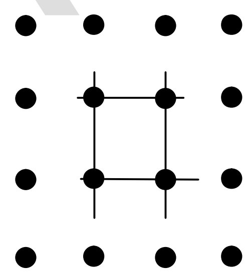
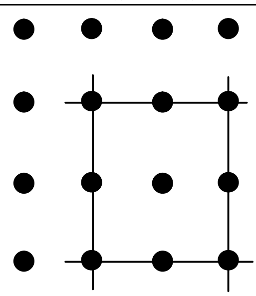
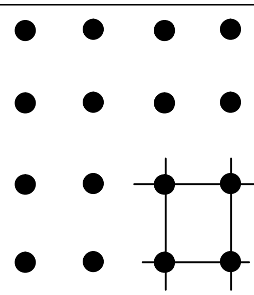
No. of coordinative Determinants = n (n – order of the matrix or Trace(A))



1 2 ... n

C_1 = Trace (A) = $\sum_{i=1}^{n} aii$ ……… (iii)

Calculation of co-factor c_2

 $\Phi_{2,1} = \begin{vmatrix} a11 & a12 \\ a21 & a22 \end{vmatrix}$	 $\Phi_{2,2} = \begin{vmatrix} a11 & a13 \\ a31 & a33 \end{vmatrix}$	 $\Phi_{2,3} = \begin{vmatrix} a11 & a1n \\ an1 & ann \end{vmatrix}$	 $\Phi_{2,4} = \begin{vmatrix} a22 & a23 \\ a32 & a33 \end{vmatrix}$
 $\Phi_{2,5} = \begin{vmatrix} a22 & a2n \\ an2 & ann \end{vmatrix}$	 $\Phi_{2,K} = \begin{vmatrix} a\,n-1,n-1 & a\,n-1,n \\ a\,n-1,n & ann \end{vmatrix}$	Where 2**K** $2,K = (n - 1)!$	

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Calculation of co-factor C_3

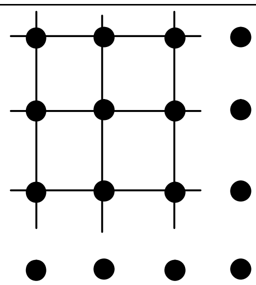
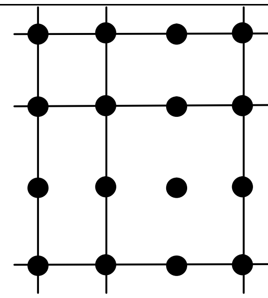
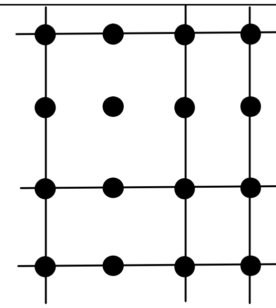
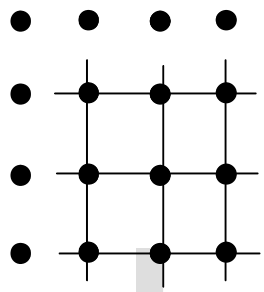
 $\Phi_{3,1} = \begin{vmatrix} a11 & a12 & a31 \\ a21 & a22 & a32 \\ a13 & a23 & a33 \end{vmatrix}$	 $\Phi_{3,2} = \begin{vmatrix} a11 & a12 & a1n \\ a21 & a22 & a2n \\ an1 & an2 & ann \end{vmatrix}$	 $\Phi_{3,3} = \begin{vmatrix} a11 & a1,n-1 & a1n \\ an-1,n & an-1,n-1 & an-1,n \\ an1 & an,n-1 & ann \end{vmatrix}$
 $\Phi_{3,K} = \begin{vmatrix} an-2,n-2 & an-2,n-1 & an-2,n \\ an-1,n-2 & an-1,n-1 & an-1,n \\ an,n-2 & an,n-1 & ann \end{vmatrix}$		Where 3,**K** $_{3,K} = (n - 2)!!_{++}$

Fig.4 Formation sub matrices with diagonal elements and determinants.

J-K sub matrix Equation

$$\Phi_{S,K} = (n - (S-1))_{S-1}!_{+} \quad \ldots\ldots\ldots\ldots\ldots \quad (iv)$$

The limits of S, $2 \leq S \leq n$

Total no. of sub matrices

$$\sum_{s=2}^{n} {}_{\Phi S,K} (n - (S-1))_{s-1}!_{+} \quad . \quad \ldots\ldots\ldots\ldots\ldots \quad (v)$$

Every sub matrix should be formed with diagonal elements and respective horizontal and vertical coordinate elements only.

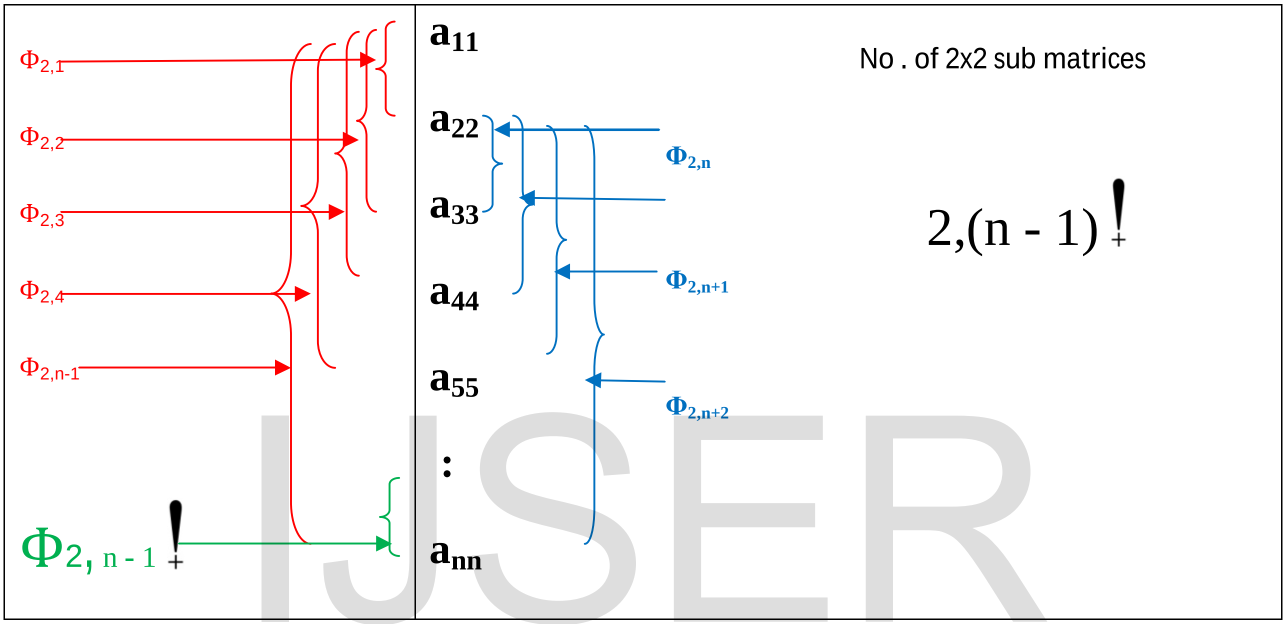
No. of diagonal elements of a sub matrix is equal to the order of the sub matrix

III. J-K chart[3] with diagonal elements for Sub matrices

Matrix – A has order of nxn i.e., n diagonal elements.

From note 2: We know that C_1 is Trace(A), (ii.b)

For C_2 : Chart of 2x2 Determinant sub matrices



For C_3: Chart of 3x3 Determinant sub matrices

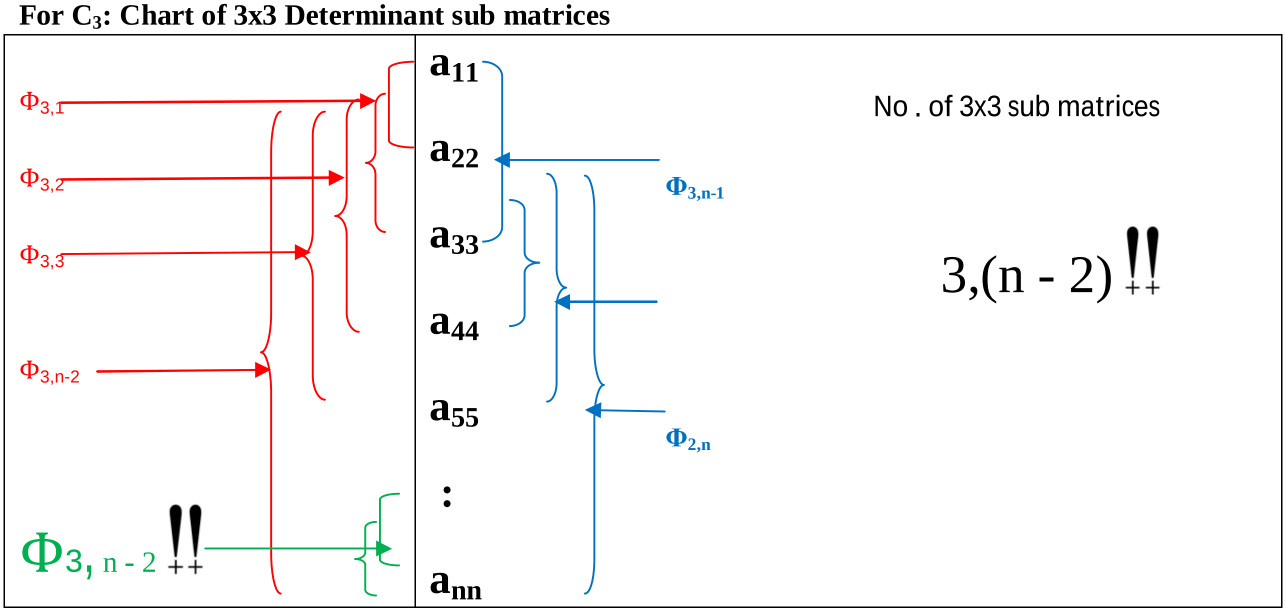


Fig .4: Formation of Determinant sub matrices of C_n

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The sign of coefficients

Notes .4: If C_0 is positive, then $\mathbf{C_n = (-1)^S}$ …. **(vi.a)**

Notes .5: If C_0 is negative, then $\mathbf{C_n = (-1)^{S+1}}$ …. **(vi.b)**

Limits of n, $0 \leq n \leq$ maximum order.

IV. Problem with Explanation

Let a matrix – A order 5x5.

$$A=\begin{bmatrix} 5 & 2 & 2 & 1 & 7 \\ 9 & 5 & 9 & 8 & 3 \\ 9 & 9 & 8 & 1 & 5 \\ 6 & 5 & 6 & 3 & 7 \\ 3 & 6 & 7 & 8 & 2 \end{bmatrix}$$

The characteristic Polynomial equation of the Matrix – A is

$$|A - \lambda I| = 0$$

Where I is 5x5 Identity matrix

$$\begin{vmatrix} 5-\lambda & 2 & 2 & 1 & 7 \\ 9 & 5-\lambda & 9 & 8 & 3 \\ 9 & 9 & 8-\lambda & 1 & 5 \\ 6 & 5 & 6 & 3-\lambda & 7 \\ 3 & 6 & 7 & 8 & 2-\lambda \end{vmatrix} = 0$$

The polynomial is, $\mathbf{-\lambda^5 + 23\lambda^4 + 98\lambda^3 + 196\lambda^2 + 1099\lambda + 3017 = 0}$ **…… (a)**

a) Calculation of Characteristic Equation by using Jeevan – Kushalaiah Method

Given Matrix, $\mathbf{A} = \begin{bmatrix} 5 & 2 & 2 & 1 & 7 \\ 9 & 5 & 9 & 8 & 3 \\ 9 & 9 & 8 & 1 & 5 \\ 6 & 5 & 6 & 3 & 7 \\ 3 & 6 & 7 & 8 & 2 \end{bmatrix}$

Step 1: from equations (ii.a), (ii.b) and (ii.c)

Notes .1: $C_0 = 1$
Notes .2: C_1 = Trace (A) = sum of diagonal elements = $\sum_{i=1}^{n} aii$
Notes .3: C_n = Determinant of Matrix-A = $|\mathbf{A}|$

$C_0 = 1$,

C_1 = Trace(A) = 5 + 5 + 8+ 3 + 2 = 23

C_n = det (A) = $|\mathbf{A}|$ = 3017

Step .2: calculate coefficients C_2 to C_{n-1}

<u>Calculation of C_2 sub matrices determinants</u>

The maximum no. of 2X2 sub matrices are, i.e., from equation (iv)

$$\Phi_{S,K} = (n - (S-1))_{S-1}{}^{!}_{+}$$

Here S = 2

$2,K = (n - 1){}^{!}_{+} = (5 - 1) = 4{}^{!}_{+} = 4+3+2+1 = 10$

$\Phi_{2,1} = \begin{vmatrix} a11 & a12 \\ a21 & a22 \end{vmatrix} = \begin{vmatrix} 5 & 2 \\ 9 & 5 \end{vmatrix} = 25 - 18 = 7$

$\Phi_{2,2} = \begin{vmatrix} a11 & a13 \\ a31 & a33 \end{vmatrix} = \begin{vmatrix} 5 & 2 \\ 9 & 8 \end{vmatrix} = 40 - 18 = 22$

$\Phi_{2,3} = \begin{vmatrix} a11 & a14 \\ a41 & a44 \end{vmatrix} = \begin{vmatrix} 5 & 1 \\ 6 & 3 \end{vmatrix} = 15 - 6 = 9$

$\Phi_{2,4} = \begin{vmatrix} a11 & a15 \\ a51 & a55 \end{vmatrix} = \begin{vmatrix} 5 & 7 \\ 3 & 2 \end{vmatrix} = 10 - 21 = -11$

$\Phi_{2,5} = \begin{vmatrix} a22 & a23 \\ a32 & a33 \end{vmatrix} = \begin{vmatrix} 5 & 9 \\ 9 & 8 \end{vmatrix} = 40 - 81 = -41$

$\Phi_{2,6} = \begin{vmatrix} a22 & a24 \\ a42 & a44 \end{vmatrix} = \begin{vmatrix} 5 & 8 \\ 5 & 3 \end{vmatrix} = 15 - 40 = -25$

$\Phi_{2,7} = \begin{vmatrix} a22 & a25 \\ a52 & a55 \end{vmatrix} = \begin{vmatrix} 5 & 3 \\ 6 & 2 \end{vmatrix} = 10 - 18 = -8$

$\Phi_{2,8} = \begin{vmatrix} a33 & a34 \\ a43 & a44 \end{vmatrix} = \begin{vmatrix} 8 & 1 \\ 6 & 3 \end{vmatrix} = 24 - 6 = 18$

$\Phi_{2,9} = \begin{vmatrix} a33 & a35 \\ a53 & a55 \end{vmatrix} = \begin{vmatrix} 8 & 5 \\ 7 & 2 \end{vmatrix} = 16 - 35 = -19$

$\Phi_{2,10} = \begin{vmatrix} a44 & a45 \\ a54 & a55 \end{vmatrix} = \begin{vmatrix} 3 & 7 \\ 8 & 2 \end{vmatrix} = 6 - 56 = -50$

<u>Calculation of C_3 sub matrices determinants</u>

The maximum no. of 2X2 sub matrices are

Here S = 3, substituting in equation (iv)

$3,K = (n - 2){}^{!}_{+} = (5 - 2) = 3{}^{!!}_{++} = 3{}^{!}_{+}+2{}^{!}_{+}+1{}^{!}_{+} = 3+2+1+2+1+1 = 10$

$\Phi_{3,1} = \begin{vmatrix} 5 & 2 & 2 \\ 9 & 5 & 9 \\ 9 & 9 & 8 \end{vmatrix} = -115$

$\Phi_{3,2} = \begin{vmatrix} 5 & 2 & 1 \\ 9 & 5 & 8 \\ 6 & 5 & 3 \end{vmatrix} = -68$

$\Phi_{3,3} = \begin{vmatrix} 5 & 2 & 7 \\ 9 & 5 & 3 \\ 3 & 6 & 2 \end{vmatrix} = 215$

$\Phi_{3,4} = \begin{vmatrix} 5 & 2 & 1 \\ 9 & 8 & 1 \\ 6 & 6 & 3 \end{vmatrix} = 54$

$\Phi_{3,5} = \begin{vmatrix} 5 & 2 & 7 \\ 9 & 8 & 5 \\ 3 & 7 & 2 \end{vmatrix} = 172$

$\Phi_{3,6} = \begin{vmatrix} 5 & 1 & 7 \\ 6 & 3 & 7 \\ 3 & 8 & 2 \end{vmatrix} = 32$

$\Phi_{3,7} = \begin{vmatrix} 5 & 9 & 8 \\ 9 & 8 & 1 \\ 5 & 6 & 3 \end{vmatrix} = 4$

$\Phi_{3,8} = \begin{vmatrix} 5 & 9 & 3 \\ 9 & 8 & 5 \\ 6 & 7 & 2 \end{vmatrix} = 58$

$\Phi_{3,9} = \begin{vmatrix} 5 & 8 & 3 \\ 5 & 3 & 7 \\ 6 & 8 & 2 \end{vmatrix} = 72$

$\Phi_{3,10} = \begin{vmatrix} 8 & 1 & 5 \\ 6 & 3 & 7 \\ 7 & 8 & 2 \end{vmatrix} = -228$

<u>Calculation of C_4 sub matrices determinants</u>

The maximum no. of 2X2 sub matrices are

Here S = 4, substituting in equation (iv)

4,K = (n - 3)! = (5 - 3) = 2!!! = 2!!+1!! = 2! + 1! +1 ! = 2 + 1 + 1 + 1 = 5

$$\Phi_{4,1} = \begin{vmatrix} 5 & 2 & 2 & 1 \\ 9 & 5 & 9 & 8 \\ 9 & 9 & 8 & 1 \\ 6 & 5 & 6 & 3 \end{vmatrix} = 5$$

$$\Phi_{4,2} = \begin{vmatrix} 5 & 2 & 2 & 7 \\ 9 & 5 & 9 & 3 \\ 9 & 9 & 8 & 5 \\ 3 & 6 & 7 & 2 \end{vmatrix} = -913$$

$$\Phi_{4,3} = \begin{vmatrix} 5 & 2 & 1 & 7 \\ 9 & 8 & 1 & 5 \\ 6 & 6 & 3 & 7 \\ 3 & 7 & 8 & 2 \end{vmatrix} = -474$$

$$\Phi_{4,4} = \begin{vmatrix} 5 & 2 & 1 & 7 \\ 9 & 5 & 8 & 3 \\ 6 & 5 & 3 & 7 \\ 3 & 6 & 8 & 2 \end{vmatrix} = -495$$

$$\Phi_{4,5} = \begin{vmatrix} 5 & 9 & 8 & 3 \\ 9 & 8 & 1 & 5 \\ 5 & 6 & 3 & 7 \\ 6 & 7 & 8 & 2 \end{vmatrix} = 778$$

$C_2 = \sum_i^{(n-1)!} \Phi_{2,i} = \Phi_{2,1} + \Phi_{2,2} + \Phi_{2,3} + \Phi_{2,4} + \Phi_{2,5} + \Phi_{2,6} + \Phi_{2,7} + \Phi_{2,8} + \Phi_{2,9} + \Phi_{2,10}$ = 7 + 22 + 9 - 11 -41 -25 -8 + 18 -19 -50 = -98

$C_3 = \sum_i^{(n-2)!!} \Phi_{3,i} = \Phi_{3,1} + \Phi_{3,2} + \Phi_{3,3} + \Phi_{3,4} + \Phi_{3,5} + \Phi_{3,6} + \Phi_{3,7} + \Phi_{3,8} + \Phi_{3,9} + \Phi_{3,10}$ = -115 - 68 + 215 + 54 +172 + 32 + 4 + 58 + 72 -228 = 196

$C_4 = \sum_i^{(n-3)!!!} \Phi_{4,i} = \Phi_{4,1} + \Phi_{4,2} + \Phi_{4,3} + \Phi_{4,4} + \Phi_{4,5}$ =5 -913 -474 -495 + 778 = -1099

Coefficients with signs:

From note 1, 2, 3 and 5

C_0 is negative = $\mathbf{(-1)^{S+1}}$ = -1

$C_1 = (-1)^{1+1} * 23 = 23$, $C_2 = (-1)^{2+1} * -98 = 98$, $C_3 = (-1)^{3+1} * 196 = 196$, $C_4 = (-1)^{4+1} * 1099 = 1099$, $C_5 = (-1)^{5+1} * 3017 = 3017$

Substituting C_0,C_1, C_2, C_3 C_4 and C_5 in equation (ii) We get

$$\mathbf{-\lambda^5 + 23\lambda^4 + 98\lambda^3 + 196\lambda^2 + 1099\lambda + 3017 = 0} \quad \text{..... (b)}$$

i.e., (a) = (b)

Hence the method id verified successfully.

The general Formula of Coefficients[4]

$$C_S = (-1)^S \sum_{\substack{s=0 \\ k=1}}^{\substack{k=(n-(S-1))S-1! \\ s=n}} \Phi_{s,k} \quad \ldots\ldots\ldots\ldots\ldots \quad (\text{vii})$$

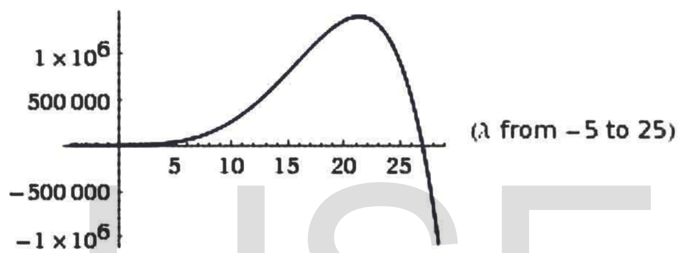
V. Result

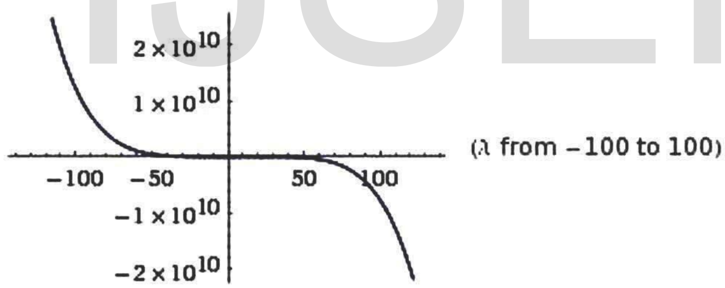
characteristic polynomial	$\begin{pmatrix} 5 & 2 & 2 & 1 & 7 \\ 9 & 5 & 9 & 8 & 3 \\ 9 & 9 & 8 & 1 & 5 \\ 6 & 5 & 6 & 3 & 7 \\ 3 & 6 & 7 & 8 & 2 \end{pmatrix}$	variable	λ

Result:

$$-\lambda^5 + 23\,\lambda^4 + 98\,\lambda^3 + 196\,\lambda^2 + 1099\,\lambda + 3017$$

Plots:





Alternate forms:

$$\lambda\,(\lambda\,(\lambda\,(98 - (\lambda - 23)\,\lambda) + 196) + 1099) + 3017$$

$$\lambda\,(\lambda\,(\lambda\,((23 - \lambda)\,\lambda + 98) + 196) + 1099) + 3017$$

Fig .5: Result calculated by Wolfram alpha (Oxford)
Direct Link of the problem with wolfram alpha:
http://www.wolframalpha.com/input/?i=characteristic+polynomial+%7B%7B5%2C2%2C2%2C1%2C7%7D%2C%7B9%2C5%2C9%2C8%2C3%7D%2C%7B9%2C9%2C8%2C1%2C5%7D%2C%7B6%2C5%2C6%2C3%2C7%7D%2C%7B3%2C6%2C7%2C8%2C2%7D%7D

VI. Conclusion

The Jeevan – Kushalaiah Method is used for calculating cofactors of any order square matrix characteristic polynomial equation to find the Eigen values and Eigen vector without using equation no. 1. This is a method based on determinants only. This method is not applied for non square matrices. The calculation and computational time very small for lower order square matrices and as the order of the square matrix increases the computational time increases. It 100 percent applicable method in real world based computational time.

VII. References

[1] Blanchard, M. M. (1928) The Basketry Book. New York: Charles Scribner's Sons

[2] Bobart, H. H. (1936) Basket work Through the Ages. London: Oxford University Press

[3] W.B Jurkat and H.G Ryser, “Matrix factorizasation and Determinants of permanents”, J algebra 3,1966,1-27 Math. 1356, Springer - Verlag, 1988

[4] H. Volkmer, Multiparameter eigenvalue problems and expansion theorems. Lect. Notes.

[5] J. W. Demmel, Applied Numerical Linear Algebra, SIAM, Philadelphia, 1997

[6] L. N. Trefethen and D. Bau, III, Numerical Linear Algebra, SIAM, Philadelphia, 1997

[7] Wikipedia link: http://en.wikipedia.org/wiki/Basket_weaving

[8] Wikipedia link: https://en.wikipedia.org/wiki/Summation

[9] Wikipedia link http://en.wikipedia.org/wiki/Characteristic_polynomial

[10] T.S. Blyth & E.F. Robertson (1998) Basic Linear Algebra, p 149, Springer ISBN 3-540-76122-5 .

[11] John B. Fraleigh & Raymond A. Beauregard (1990) Linear Algebra 2nd edition, p 246, Addison-Wesley ISBN 0-201-11949-8

[12] Werner Greub (1974) Linear Algebra 4th edition, pp 120–5, Springer, ISBN 0-387-90110-8